# SYNOPTIC: Effect of Support Elasticity on the Bending of Axisymmetric Plates, S. T. Gulati, Research and Development Laboratories, Corning Glass Works, N.Y., *AIAA Journal*, Vol. 8, No. 9, pp. 1635–1638.

**Structural Static Analysis: Aircraft Structural Design; Marine Vessel Design; Atmospheric, Space and Oceanographic Sciences.**

## Theme

This paper compares the Raleigh-Ritz solution of the problem of bending of an axisymmetric plate supported elastically at the edge with the exact solution and discusses the accuracy and breakdown of the former solution.

## Content

For a plate with elastic edge supports the Raleigh-Ritz solution undergoes considerable simplification if it is assumed that the plate slope remains constant in the support region. This assumption is reasonable from the deflection and slope point of view, particularly in those cases where the support width is small compared with the plate dimensions. The exact solution, although straightforward, is computationally very cumbersome and requires the use of a computer. The validity of the assumption of constant slope in the support region is examined by considering the flexure of an axisymmetric plate, for which the Raleigh-Ritz solution is in closed form and hence facilitates the examination of the dependence of stresses and deflection on the foundation modulus and width of the support.

The solution for the inner region of the axisymmetric plate under the assumptions of Poisson-Kirchhoff plate theory involves two constants of integration;

$$\bar{w} = \rho^4/64m^4 + (\bar{A}_1/4)\rho^2 + \bar{A}_2 \tag{1}$$

For Raleigh-Ritz solution the deflection in the support region is taken as

$$\bar{w}_s = \bar{w}(m) + (\rho - m)(d\bar{w}/d\rho)(m) \tag{2}$$

The potential energy is then formulated and subsequently minimized to obtain the constants $\bar{A}_1$ and $\bar{A}_2$. The remaining symbols appearing in Eqs. (1) and (2) are: $\bar{w} = wD/q_o c^4$, dimensionless deflection; $\rho = r/a$, dimensionless radius; $m = c/a$; $q_o$ = intensity of normal load carried by the plate; $a$ = plate radius; $c$ = radius of the inner region of plate; $D$ = plate flexural rigidity; $w$ = plate deflection; and $r$ = radial coordinate.

The exact solution in the support region is given by the solution of the fourth-order equation

$$\frac{d^4w}{d\rho^4} + \frac{2}{\rho}\frac{d^3w}{d\rho^3} - \frac{1}{\rho^2}\frac{d^2w}{d\rho^2} + \frac{1}{\rho^3}\frac{dw}{d\rho} + \left(\frac{2ka^4}{D}\right) w = 0 \tag{3}$$

$$k = \text{support modulus}$$

and involves *ber*, *bei*, *ker*, and *kei* functions and four additional constants of integration. The six constants of the exact solution are determined by satisfying two boundary conditions at the free edge $r = a$ and four continuity conditions (continuity of $w$, $dw/d\rho$, $d^2w/d\rho^2$ and $d^3w/d\rho^3$) at $r = c$.

Comparison of the two solutions indicates that the assumption of constant slope in the support region is quite good as far as stresses and deflection in the inner region, and the deflection in the support region, are concerned. The stresses in the support region, however, are in error. It is therefore concluded that if the stresses in the support region are of interest, then the exact solution is imperative.